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Abstract

Linear systems of neutral type are considered using the infinite dimensional approach. Conditions for exact controllability and regular asymptotic stabilizability are given. The main tools are the moment problem approach and the existence of a Riesz basis of invariant subspaces.

Keywords

Neutral type systems, Riesz basis, exact controllability, stabilizability.

In this paper we consider the problem of controllability and stabilizability for a general class of neutral systems with distributed delays given by the equation

$$\dot{z}(t)-A_{-1}\dot{z}(t-1)=Lz_t(\cdot)=\int_{-1}^{0}A_2(\theta)\dot{z}(t+\theta)\mathrm{d}\theta+\int_{-1}^{0}A_3(\theta)z(t+\theta)\mathrm{d}\theta+Bu(t), \quad (1)$$

where A_{-1} is a constant $n\times n$-matrix, A_2, A_3 are $n\times n$, L_2 valued matrices. We consider the operator model of the neutral type system (1) in the product space $M_2=\mathbb{C}^n\times L_2(-1,0;\mathbb{C}^n)$, so (1) can be reformulated as

$$\dot{x}(t)=\mathcal{A}x(t)+\mathcal{B}u(t),\ x(0)=\begin{pmatrix} y\\ z(\cdot)\end{pmatrix},\ \mathcal{A}=\begin{pmatrix}0 & L\\ 0 & \frac{\mathrm{d}}{\mathrm{d}\theta}\end{pmatrix},\ \mathcal{B}=\begin{pmatrix}B\\ 0\end{pmatrix}, \quad (2)$$

with $\mathcal{D}(\mathcal{A})=\{(y,z(\cdot))\in M_2 : z\in H^1([-1,0];\mathbb{C}), y=z(0)-A_{-1}z(-1)\}$, and $\mathcal{A}$ is the generator of a C_0-semigroup. The reachability set $\mathcal{R}_T$ is such that $\mathcal{R}_T\subset\mathcal{D}(\mathcal{A})$ for all $T>0$, with $u(\cdot)\in L_2$, the solution of (2) being in $\mathcal{D}(\mathcal{A})$.

Theorem 1. *The system (2) is exactly null-controllable, i.e.* $\mathcal{R}_T=\mathcal{D}(\mathcal{A})$, *iff the pair* (A_{-1},B) *is controllable and* rank $\begin{pmatrix}\Delta_{\mathcal{A}}(\lambda) & B\end{pmatrix}=n$ *for all* $\lambda\in\mathbb{C}$, *where*

$$\Delta_{\mathcal{A}}(\lambda)=\lambda I-\lambda\mathrm{e}^{-\lambda}A_{-1}-\lambda\int_{-1}^{0}\mathrm{e}^{\lambda s}A_2(s)\mathrm{d}s-\int_{-1}^{0}\mathrm{e}^{\lambda s}A_3(s)\mathrm{d}s,$$

If these conditions hold then the system is controllable at any time $T > n_1$, where n_1 is the controllability index of the pair (A_{-1}, B). It is not controllable at $T \leq n_1$.

The main tools of the analysis is the moment problem approach and the theory of basis of exponential families. We construct a special Riesz basis using the existence of a Riesz basis of invariant subspaces [5] and describe the controllability problem via a moment problem in order to get the time of controllability. See [3] for the monovariable and discrete delay case, via a different approach, and [4] for a preliminary result.

The same Riesz basis of subspaces allows to characterize the problem of asymptotic stabilizability by a regular feedback law. From the operator point of view, the regular feedback law

$$u = \mathcal{F}x = \int_{-1}^{0} F_2(\theta)\dot{z}(t+\theta)\mathrm{d}t + \int_{-1}^{0} F_3(\theta)z(t+\theta)\mathrm{d}t, \tag{3}$$

where $F_2, F_3 \in L_2(-1, 0; \mathbb{C}^{n\times n})$ means a perturbation of $\mathcal{A}$ by the operator $\mathcal{BF}$ which is relatively $\mathcal{A}$-bounded and verifies $\mathcal{D}(\mathcal{A}) = \mathcal{D}(\mathcal{A} + \mathcal{BF})$. Such a perturbation does not mean, in general, that $\mathcal{A} + \mathcal{BF}$ is the infinitesimal generator of a C_0-semigroup. However, in our case, this fact is verified directly since after the feedback we get also a neutral type system like (1) with $\mathcal{D}(\mathcal{A}) = \mathcal{D}(\mathcal{A} + \mathcal{BF})$. This feedback law is essentially different from that which use the term $F\dot{x}(t-1)$ (cf. for example [2]) and for which $\mathcal{D}(\mathcal{A}) \neq \mathcal{D}(\mathcal{A} + \mathcal{BF})$. Our main result is

Theorem 2. (Rabah, Sklyar & Rezounenko) *Under the assumptions: the eigenvalues of the matrix A_{-1} satisfy $|\mu| \leq 1$, the eigenvalues $\mu_j, |\mu_j| = 1$ are simple, the system (1) is regularly asymptotically stabilizable if* $\operatorname{rank}\begin{pmatrix}\Delta_{\mathcal{A}}(\lambda) & B\end{pmatrix} = n$ *for all* $\lambda : \operatorname{Re}\lambda \geq 0$, *and* $\operatorname{rank}\begin{pmatrix}\mu I - A_{-1} & B\end{pmatrix} = n$ *for all* $\mu : |\mu| = 1$.

In the case when A_{-1} has at least one eigenvalue $|\mu| = 1$ with a nontrivial Jordan chain, the system can *not* be stabilized by a control of the form (3). The same if $\sigma(A_{-1}) \not\subset \{\mu : |\mu| \leq 1\}$. This follows from the fact that any control of the form (3) leaves the system in the same form and then it remains unstable [5].

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